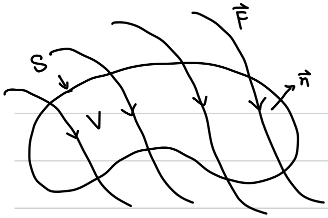
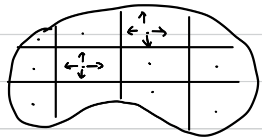


The flux of a vector field $\vec{F}$ through a closed surface S is equal to the integral of the $\text{div}(\vec{F})$ over the entire enclosed volume 'V'

$$\iiint_V \vec{\nabla}\cdot\vec{F}\, dV = \underbrace{\iint_S \vec{F}\,\vec{n}\, dS}_{\text{Flux through } S}$$



inner parts are cancelled out if we cut the grid fine enough. So what we left is the flux.

Mass conservation in volume

Total mass in $V = \iiint_V \overbrace{\rho}^{\text{density}} dV$ $\qquad \rho(x,y,z,t)$

$$\frac{d}{dt}\iiint_V \rho\, dV = -\iint_S \rho\vec{F}\,\vec{n}\, dS + \iiint_V Q\, dV. \quad \text{source}$$

$$= -\iiint_V \vec{\nabla}\cdot(\rho\vec{F})\, dV$$

$$\vec{F} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

$$\iiint_V \left(\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{F})\right) dV = 0 \qquad \text{True for all volumes } V$$

$$\Rightarrow \frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{F}) = 0 \qquad \text{mass continuity eqt.}$$

$$= Q$$

$$\vec{\nabla}\cdot(\rho\vec{F}) = \frac{\partial}{\partial x}\rho F_1 + \frac{\partial}{\partial y}\rho F_2 + \frac{\partial}{\partial z}\rho F_3$$

$$= \rho_x F_1 + \rho F_{1x} + \rho_y F_2 + \rho F_{2y} + \rho_z F_3 + \rho F_{3z}$$

$$= \rho(\vec{\nabla}\cdot\vec{F}) + (\nabla\rho)\cdot\vec{F}$$

If fluid is incompressible, then $\rho = \text{constant}$. $\frac{\partial\rho}{\partial t} = 0$ and $\nabla\rho = 0$

$$\frac{\partial\rho}{\partial t} + \rho(\vec{\nabla}\cdot\vec{F}) + (\nabla\rho)\cdot\vec{F} = 0 \quad \Rightarrow \quad \vec{\nabla}\cdot\vec{F} = 0$$

$\vec{F}\cdot\vec{n}$ measures the amount of vector field going parallel with $\vec{n}$ (flux flowing out)

Suppose Ω open $\subseteq \mathbb{R}^n$ and $\sigma: \Omega \to \mathbb{R}^n$ is diff'ble at every pt of Ω. If f is a real valued function defined on $\sigma(\Omega)$, then $\int_{\sigma(\Omega)} f(u)\,du = \int_\Omega f(\sigma(x))\,|\det \sigma'(x)|\,dx$